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CRITERION FOR THE ONSET OF CONVECTIVE FLOW IN A FLUID IN A POROUS MEDIUM

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Abstract—The purpose of the present paper is to make a careful study of the breakdown of stability of a layer of fluid subject to a vertical temperature gradient in a porous medium, and to give a conclusive cirterion for the onset of convection currents.

Through theoretical examinations, it is pointed out that the phenomenon is affected with a specially defined thermal diffusivity, and that there are possibilities for the ordinary theory based upon Darcy's law to be applicable even when the permeability of the porous medium becomes considerably high. Careful experiments also are carried out, in which the difficulties to generate the convective flow in the porous medium under reasonable temperature gradient are overcome by the use of the porous medium of comparatively high permeability, as well as by the use of a compressible gas as the fluid.

Satisfying agreement of the experimental results with the theory is obtained to provide conclusions that the criterion for the onset of convective flow is certainly given by the equation: $Ra \,.\, k/l^2 = 4\pi^2$ where Ra is Rayleigh number, k the permeability, and l the vertical thickness of the porous medium, but that the thermal diffusivity included in Ra must be defined as the thermal conductivity of porous medium divided by the specific heat capacity of fluid.

NOMENCLATURE

- c_p, specific heat at constant pressure;
- d, diameter of filling particles;
- g, gravitational acceleration;
- k, permeability;
- l, vertical thickness of porous layer;
- p, pressure of fluid;
- Ra, Rayleigh number, $= g\beta l^3 \Delta T/(\nu\varkappa)$;
- T, temperature;
- ΔT, temperature difference between upper and lower bounding plates;
- t, time;
- u, v, w, components of velocity;
- x, y, z, coordinates.

Greek symbols

- β, cubical expansion-coefficient;
- ε, porosity;
- $\varkappa$, thermal diffusivity;
- λ, thermal conductivity;
- λ_m, thermal conductivity of porous medium (without convection);
- μ, viscosity;
- ν, kinematic viscosity;
- ρ, density;
- $(\ )_f$, value of fluid;
- $(\ )_m$, value of mixture of solid and fluid;
- $(\ )_s$, value of solid.

1. INTRODUCTION

THE PROBLEM of the occurrence of convection currents in a horizontal layer of viscous fluid such as shown in Fig. 1(a) has been given a conclusive answer by experimental verifications, as well as by the theory originated by Rayleigh and finally extended to the work of Pellew and Southwell [1]. On the other hand, Fig. 1(b) is a porous medium composed with spherical fillings as an example. The convective flow occurs in such a case also when heated from below, but the criterion for the occurrence of convection has not been confirmed so well as in the former case.

In connection with the distribution of NaCl in subterranean sand-layers, Horton and Rogers [2] made a theoretical analysis. Lapwood [3] also solved the problems of similar kind independently and more exactly, and gave the same result as that of Horton and Rogers for the porous medium which is bounded above and below by rigid and conducting boundaries. On the other hand, Morrison, Rogers and Horton [4] made experiments to verify the theoretical result, but due to the unsteady conditions under which their experiments were made as well as due to other reasons, not only were the results complicated but they did not agree with the theoretical predictions. Then Rogers and others [5, 6] attempted to compare their experiments with the approximate theories which allow for the non-linear temperature distributions and for the temperature dependence of viscosity, but it cannot be denied from the viewpoint of the fundamental analysis of the phenomenon that great ambiguities are left.

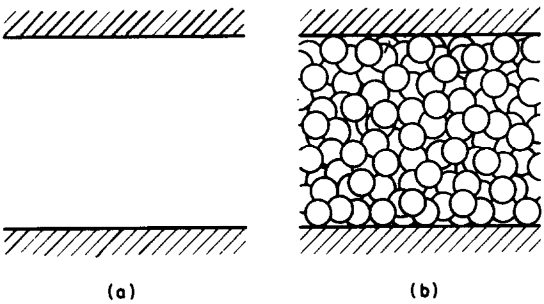

FIG. 1.
(a) Horizontal layer of fluid.
(b) Porous layer with spherical fillings.

At this stage, the present work has been attempted to give a definite answer to the problem through both the examinations of the theory and experimental verifications.

2. EXAMINATIONS OF THEORY

The actual flow of fluid in a porous medium is replaced by the hypothetical uniform flow of the same gross rate, assuming the space to be homogeneous. For this flow the expression of the resistance differs from the ordinary flow, and instead of the shearing stress proportional to the velocity gradient, Darcy's law is generally applied:

$$(\mu/k)\mathbf{v} = -\operatorname{grad} p \tag{1}$$

where μ is the viscosity, k the permeability (a constant with the dimension of square of length), $\mathbf{v}$ the macroscopic velocity and p the pressure of the fluid.

Taking rectangular axes in a porous medium as shown in Fig. 2 where the space is assumed to be uniform, let u, v and w be the macroscopic velocity components of fluid, and let ρ be the density, T the temperature, t the time, and g the gravitational acceleration. Then, only replacing the customary viscosity term in the equations of a horizontal layer of fluid by the resistance term in the left-hand side of (1), provides the equations of continuity, of motion and of energy as follows:

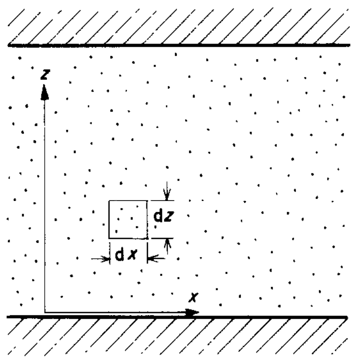

FIG. 2. Porous layer bounded by two horizontal surfaces.

$$-\frac{\mathrm{D}\rho}{\mathrm{D}t} = \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \tag{2}$$

$$\rho\frac{\mathrm{D}}{\mathrm{D}t}(u, v, w) = (0, 0, -\rho g) - \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)p - \frac{\mu}{k}(u, v, w) \tag{3}$$

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \varkappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

where $\varkappa$ in (4) is the thermal diffusivity of porous medium.

The criterion for the onset of convection currents can be derived with these governing equations in quite the same way as in the horizontal layer of fluid, except that the boundary conditions for velocity at the upper and lower bounding planes are now $w = 0$ instead of $u = v = w = 0$. Then the criterion, which Lapwood [3] has derived, can be written as

$$Ra \cdot \frac{k}{l^2} = 4\pi^2 \tag{5}$$

where Rayleigh number $Ra \equiv g\beta l^3 \Delta T/(\nu\varkappa)$, and β is the cubical expansion-coefficient of fluid, l the vertical thickness of porous medium, ΔT the temperature difference between the two bounding planes, and ν the kinematic viscosity.

2.1 *Examinations*

Since the fillings are included in a small control volume shown in Fig. 2, $\partial\rho/\partial t$ included in $D\rho/Dt$ in (2) must be correctly replaced by $\varepsilon\partial\rho/\partial t$, the ε denoting the porosity (Muskat [7]). In the left-hand side of (3), the forces of inertia depending upon the time variation of the macroscopic velocities are different from the actual forces. In addition, it is used to neglect the force of inertia in the flow through a porous medium, since the force of inertia is extremely small as compared with the viscous resistance.

These points, however, go out of the question for the following circumstances. In the theoretical investigation on the onset of convection currents, (a) variations of density are used to be neglected, except in so far as they modify the action of gravity; (b) second-order terms are used to be neglected; (c) as pointed out primarily by Jeffreys [8], the onset of convection currents is characterized by the condition of marginal stability, that is, the condition which is obtained by placing $\partial/\partial t = 0$. These three conditions, which are quite reasonable, make the left-hand sides of both (2) and (3) vanish to leave no space for the questions aforementioned to affect the theoretical prediction of (5).

However, the balance among the time variation of the energy stored in a small control volume shown in Fig. 2, the transportation of enthalpy by the flow of fluid, and the heat conduction through the control volume, yields

$$(c_p\rho)_m \frac{\partial T}{\partial t} + (c_p\rho)_f \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = \lambda_m \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{6}$$

where c_p is the specific heat at constant pressure, ρ the density, and λ the thermal conductivity. The suffix m denotes the mixture of solid fillings and fluid, and the suffix f denotes the fluid.

Comparing (4) with (6), and placing $\partial/\partial t = 0$ from the condition of marginal stability, it is found that the special thermal diffusivity defined as

$$\varkappa = \lambda_m/(c_p\rho)_f \tag{7}$$

must be used in (5) instead of

$$\varkappa = \lambda_m/(c_p\rho)_m \tag{8}$$

which has been used up to now. Since the magnitude of $\varkappa$ defined in (7) is considerably different from that in (8) in general, the distinction between them is of great importance.

2.2 *Applicable range of equation* (5)

The inclined solid line shown in Fig. 3 is the result of (5), giving the critical Rayleigh number Ra as a function of k/l^2. When the

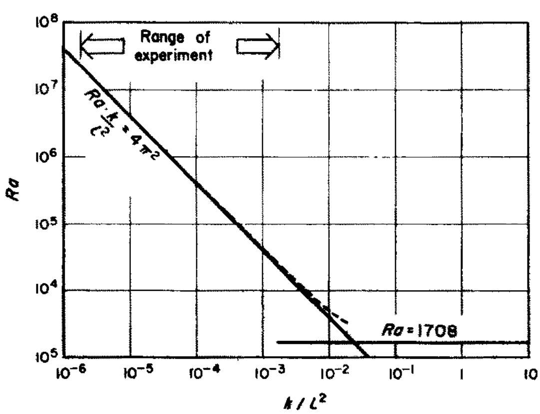

FIG. 3. Critical Rayleigh number Ra as a function of k/l^2.

permeability k becomes high, however, the resistance due to Darcy's law (1) inevitably reduces. It is, therefore, presumed from the physical point of view that the phenomenon changes its mode towards that of the ordinary horizontal layer of fluid, the horizontal line in Fig. 3 showing its theoretical criterion of $Ra = 1708$.

Although the exact analysis of this transitional region has great difficulties, let us attempt the following analysis for the sake of convenience. In the governing equations (2), (3) and (4), the ordinary viscous resistance $\mu\nabla^2(u, v, w)$ is added to the right-hand side of (3) yielding

$$\rho\frac{D}{Dt}(u, v, w) = (0, 0, -\rho g) - \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)p - \frac{\mu}{k}(u, v, w) + \mu\nabla^2(u, v, w) \qquad (3')$$

where the boundary conditions for velocity at two bounding planes are $u = v = w = 0$. As clear from (3′), the present analysis approaches to the theory of the porous medium as the permeability k reduces, and it does to the theory of the ordinary layer of fluid as k increases; consequently, as the first approximation at least, the present analysis can connect the two theories.

The analysis is accomplished in the way analogous to that of Pellew and Southwell [1]. Under the conditions of (a), (b) and (c) described in Section 2.1, the small departure from the state of equilibrium are considered following (2), (3′) and (4). Eliminating u and v as well as the departure of temperature and pressure by combining the equations, yields the following differential equation as to the vertical component of velocity w:

$$\left(\nabla^2 - \frac{1}{k}\right)\nabla^4 w + \frac{g\beta\alpha}{\nu\varkappa}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \qquad (9)$$

α denoting the temperature gradient ($\alpha < 0$).

Putting $w = w(z)\sin mx \sin ny$ and $z = l\zeta$ in (9), yields

$$\left[(D^2 - a^2)^3 - \frac{(D^2 - a^2)^2}{k/l^2} + Ra\,.\,a^2\right] w = 0 \qquad (10)$$

where we have written $D \equiv \partial/\partial\zeta, a^2 \equiv (m^2 + n^2)l^2$ which is a characteristc number not yet determined, and $Ra = -g\beta l^4\alpha/(\nu\varkappa) = g\beta l^3\Delta T/(\nu\varkappa)$ which is Rayleigh number. The boundary conditions at the two rigid conducting planes that $u = v = w = 0$ and no disturbance of temperature, also can be unified in terms of w as:

$$\left.\begin{aligned} &w = 0, \\ &Dw = 0, \\ &[(D^2 - a^2)^2 - (D^2 - a^2)/(k/l^2)]\,w = 0. \end{aligned}\right\} \qquad (11)$$

The general solution of (10) can be written in the form

$$w = A_1\cosh 2\gamma_1\zeta + A_2\cosh 2\gamma_2\zeta + A_3\cosh 2\gamma_3\zeta + B_1\sinh 2\gamma_1\zeta + B_2\sinh 2\gamma_2\zeta + B_3\sinh 2\gamma_3\zeta \qquad (12)$$

where A_1, A_2, A_3, B_1, B_2, B_3 are arbitrary and $4\gamma_1^2$, $4\gamma_2^2$, $4\gamma_3^2$ are the three values of D^2 which are given when (10) is solved symbolically. If k/l^2 is small as in the case of particular importance in the present paper, all the three values of D^2 are real and they are

$$\left.\begin{aligned} &D^2 = 4\gamma^2 = a^2 + \frac{1 + 2\cos r/3}{3k/l^2}; \\ &a^2 + \frac{1 + 2\cos(r/3 + 120^\circ)}{3k/l^2}; \\ &a^2 + \frac{1 + 2\cos(r/3 + 240^\circ)}{3k/l^2} \\ &\text{where} \\ &\cos r = 1 - \tfrac{1}{2}\,Ra\,.\,a^2(3k/l^2)^3. \end{aligned}\right\} \qquad (13)$$

If k/l^2 is sufficiently small, (13) can be approximated as follows:

$$4\gamma^2 = a^2 + 1/(k/l^2); \qquad a^2 - a\sqrt{(Ra\,.\,k/l^2)}; \qquad a^2 + a\sqrt{(Ra\,.\,k/l^2)}.$$

The six arbitrary constants in (12) are determined so as to satisfy the three boundary conditions of (11) at each of the upper and lower boundaries. The problem is to find the state permitting the convective flow, that is, the state for which the six constants in (12) do not vanish simultaneously. For convenience the origin of ζ will now be changed so as to define the upper and lower boundaries at $\zeta = \frac{1}{2}$, $\zeta = -\frac{1}{2}$ respectively. Then, in order for (12) to satisfy (11) at $\zeta = \pm\frac{1}{2}$ simultaneously, it should be either $B_1 = B_2 = B_3 = 0$ (even solution) or $A_1 = A_2 = A_3 = 0$ (odd solution).

Substitution of the even solution into (11) gives three conditional equations for A_1, A_2, A_3, leading us to a determinant that must be satisfied to deny simultaneous vanishing of A_1, A_2, A_3. Included in the determinant are Ra, a^2 and k/l^2, so that it gives a relation between Ra and a^2 fixing k/l^2. Then the lowest admissible value of Ra can be found at any value of k/l^2 by changing a^2. The same procedure is possible for the odd solution as well, but the lowest value of Ra thus obtained is higher than that obtained with the even solution. Consequently the critical Rayleigh number is determined by the even solution, and the results are shown in Fig. 3 with a broken curve.

This theoretical presumption seems likely to suggest that the region, over which (5) is applicable, can extend at least up to the magnitude of k/l^2 of 10^{-3} or so. If the porous medium made of spherical particles is considered, $k/l^2 \doteqdot 10^{-3}$ corresponds to considerably high values of d/l, where d is the diameter of the fillings and l the vertical thickness of the porous medium.

For high values of d/l, the assumption of the homogeneity in the field may become questionable. As regards the theory, however, it must be pointed out that the microscopic state has been homogenized assuming the macroscopic velocities: u, v, w and macroscopic temperature: T, and that the theory has been constructed using the permeability k and the thermal conductivity of the porous medium λ_m which are both defined for the macroscopic quantities aforementioned. In addition, approximately speaking, the k and λ_m are to be determined as properties of the unit space per one of the filling particles. Consequently it is not unreasonable to presume that even when d/l becomes comparatively high, the accuracy of the theoretical predictions does not deteriorate excessively if very severe variations of velocity and temperature do not appear in the field.

3. EXPERIMENTAL APPARATUS

3.1 *General principles*

In comparison with the ordinary horizontal layer of fluid, the convection does not arise so well in the porous medium unless the temperature difference between the upper and lower boundaries becomes very high. If liquid is used as the fluid, it is easier than gas at the atmospheric pressure to generate the convective flow, but the necessary temperature differences are still high so that the temperature dependence of physical properties is apt to rise a discussion.

Great thickness of the porous layer also facilitates the occurrence of convection, but it takes a very long time until a steady state is accomplished. In addition, horizontal extension of the porous layer also is necessary to prevent errors due to the side effects. This is apt to generate difficulties in securing the horizontal uniformities of temperature and of contact at the upper and lower boundaries as well as in preparing plenty of the precise fillings which is necessary in the present work.

In the present study, therefore, the thickness of the porous layer is kept small, but the onset of convection is facilitated by using the filling particles of comparatively great diameter. In addition, a gas (nitrogen) is adopted as the fluid so that convection currents can be generated under low temperature differences by compressing the gas. As both the kinematic viscosity and thermal diffusivity of the gas are nearly in inverse proportion to the pressure, rapid increase of Rayleigh number follows the increase of the pressure, enabling to serve our purpose readily. Besides this, gas has two more advantages: (a) the temperature dependence of physical properties is little; (b) the heat capacity is very little as compared with that of a solid so that the difference between (7) and (8) becomes particularly great.

3.2 *Description of apparatus*

Measurements of the apparent thermal conductivity of porous layers are made under steady conditions utilizing the so-called comparison method. The onset of convection is readily found out by the sudden change of the apparent thermal conductivity.

The most essential part of the experimental apparatus is shown in Fig. 4. In a pressure vessel is a porous layer of 100 mm in diameter and of l (variable; 9 mm as the standard) in thickness set up. The upper boundary of the porous layer is in contact with a copper plate, which is cooled uniformly by the cooling water flowing over it. The lower boundary of the porous layer is in contact with a standard plate made of glass (8·1 mm in thickness and of known thermal conductivity). This standard plate is placed on another copper plate, which is heated uniformly by an electric heater placed below. Around the circumference of the porous layer, a thin frame made of Bakelite keeps the fillings.

Nitrogen was used as the fluid, and it was supplied to the pressure vessel from a gas bomb of about 150 atm in pressure. As the gas enters readily into the porous layer also, the pressure of the gas in the porous layer is the same as that in the vessel, and was measured with a calibrated Bourdon-tube pressure gauge.

The pressure vessel has nine pairs of special terminals for copper–constantan thermocouples, which were used to measure temperatures of three surfaces: the lower surface of the cooling copper plate, upper surface of the standard plate, and upper surface of the heating copper plate. Liquid paraffin was put between the standard plate and the heating copper plate not to leave a gas film there. The horizontal uniformity of temperature was found to be sufficient over each of the three surfaces. Adjusting power input to the electric heater, the temperature difference across the porous layer was kept at about 40 degC as the standard and occasionally at about 60 degC and 80 degC.

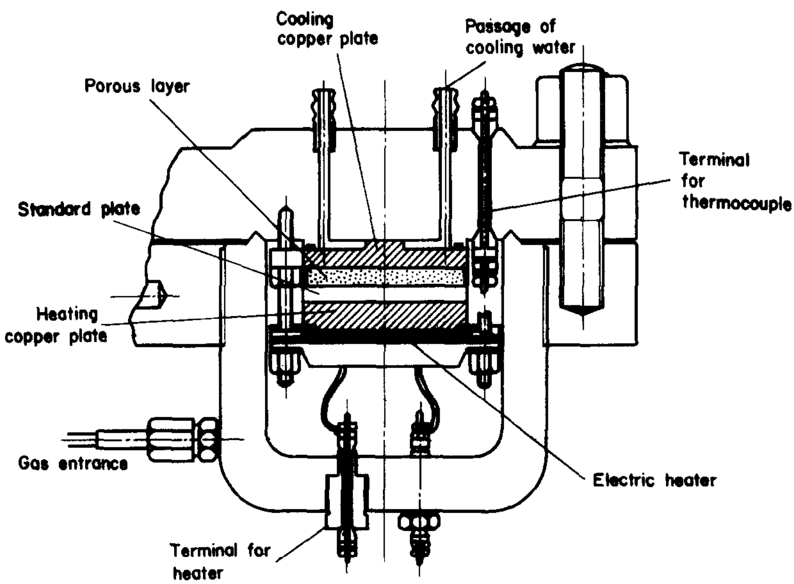

FIG. 4. Essential part of experimental apparatus.

In determining the thermal conductivity of a porous layer with the apparatus shown in Fig. 4, errors arise due to the circumferential surfaces of both the porous layer and standard plate which are exposed to the surroundings. However, theoretical examinations show that the effects are less than several percent within conditions of the present study.

3.3 *Examination of synthetic accuracy*

With an absolute method, careful measurements of the thermal conductivity of the standard plate were made within the necessary range of temperature.

Then the synthetic accuracy of the experimental apparatus was examined by measuring the apparent thermal conductivity λ of the ordinary layer of fluid, and by comparing the results with Silveston's correlation [9]. The layer of fluid was readily prepared in the apparatus without filling the particles. Temperatures of the upper and lower boundaries of the layer were kept at nearly constant respectively, and Rayleigh number Ra was changed by changing the pressure of gas only. Subtraction of effects of radiant heat transfer from the observed results was made with ease, since the temperature of each of the two boundaries was nearly constant.

Experimental results are shown in Fig. 5, where the ordinates are the ratios of λ measured to the thermal conductivity of fluid λ_f, and the solid curve is Silveston's correlation. According to Fig. 5, it may be concluded that the synthetic accuracy of the experimental apparatus is sufficient for the present study.

4. POROUS MEDIA USED

4.1 *Filling particles*

The porous media were composed with each of glass, steel and aluminium balls in order to give drastic variation of the thermal conductivity of the solid particles.

Four kinds of glass balls of 0·779 mm, 1·25 mm, 2·27 mm, and 4·66 mm in mean diameter respectively were used. Though being sold as the precision glass balls, they were found to have irregularity of shape to some extent, and the dispersion of diameters also was considerable as shown in Fig. 6.

On the other hand, belonging to the precision grade for the ball bearings, two kinds of steel balls of 2·00 mm and 4·00 mm in diameter, which were used, had almost perfect roundness and accuracy of dimension. Aluminium balls of 3·00 mm in diameter which were used, also had a good roundness and accuracy of dimension with only minor errors of dimension less than ± 1 percent.

However, being constructed as a random assemblage of filling particles, the porous medium has, in general, statistical characters so that it is governed with various kinds of mean values, and slight dispersions of the shape and dimension do not come into particular question.

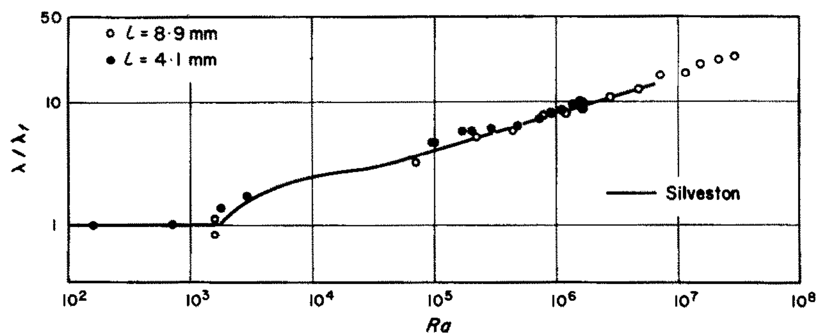

FIG. 5. Apparent thermal conductivity of horizontal layer of fluid.

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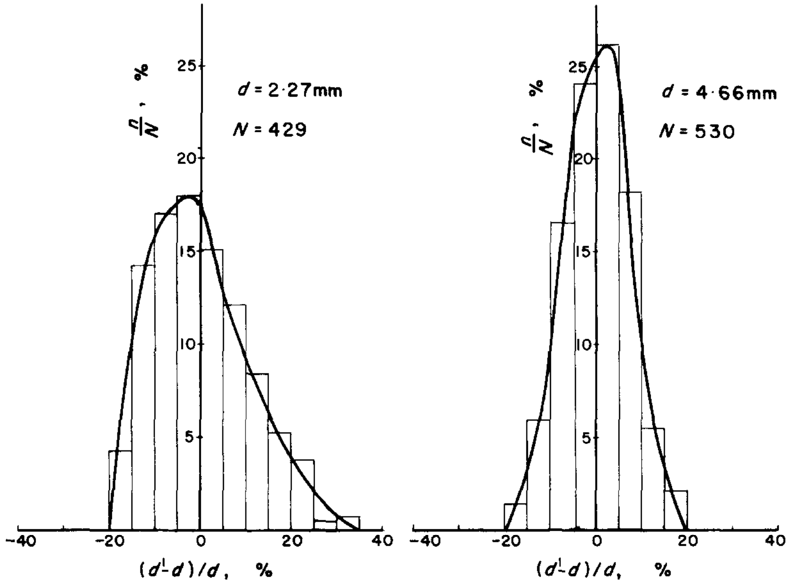

FIG. 6. Two examples showing dispersion of diameter in glass balls (d': actual diameter, d: mean diameter, n: number of samples, N: total number of samples).

Experiments were made over a comparatively wide range of $d/l = 0{\cdot}048$–1, the d being the diameter of the filling particles, and the l being the vertical thickness of the porous layer. Here, $d/l = 1$ is a very special case that mono-layer of particles is placed between the upper and lower bounding surfaces.

4.2 *Porosity*

Porosity, that is the ratio of the pore space to the whole volume of the porous medium, generally differs every time the porous layer is made. Consequently, whenever a new porous layer was submitted to the experiments, its porosity was carefully determined by measuring the weight and volume of the porous layer.

As shown in Fig. 7, the porosity ε has a tendency to increase with d/l to some extent when d/l approaches unity, and this is due to the pore space near the upper and lower bounding surfaces which is about 20 per cent greater than that inside the porous layer. However, the local difference of the pore space is not so great that its effects on the mean porosity are substantially negligible if d/l is less than, say, 0·2.

4.3 *Thermal conductivity*

In the experiment with the apparatus shown in Fig. 4, the thermal conductivity of the porous medium λ_m is necessarily measured every time the experiment is made.

It can be presumed that the effects of the

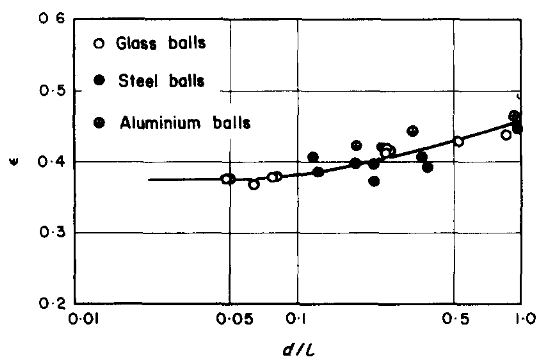

FIG. 7. Variation of porosity ε with d/l.

radiant heat transmission on λ_m are substantially neglected under conditions of the present study. Fig. 8 is the comparison of λ_m measured in the present study with λ_m estimated by the semi-empirical equations of Yagi and others [10], showing a rough agreement between them. Now, examining the rates of the radiant heat transmission included in the estimated values of λ_m, they are negligible for the steel and aluminium balls, and are less than 5 per cent even for the glass balls with a high emissivity if $d/l < 0{\cdot}2$.

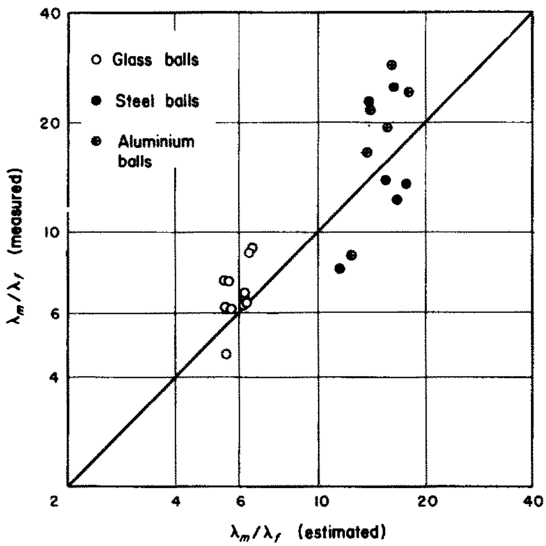

FIG. 8. Comparison of λ_m measured with λ_m estimated by semi-empirical equations of Yagi and others [10] (λ_f: thermal conductivity of fluid).

As is well known, the mechanical compression of the porous medium affects the thermal conductivity λ_m especially when the material of the filling particles has comparatively great elasticity and high thermal conductivity. The cooling copper plate in the pressure vessel shown in Fig. 4 has a tendency to slightly distort upwards as the gas pressure inside the vessel is increased, since the cooling water contacting the upper surface of the copper plate is at the atmospheric pressure. Then the precompression which has been given initially is reduced decreasing the thermal conductivity of the porous medium.

Finally it should be mentioned that the thermal conductivity of the porous medium λ_m considerably varies with the porosity when the difference of the thermal conductivity is very great between the solid particles and the fluid. Fig. 9 shows the variation of λ_m with d/l, for which it should be reminded that the mean porosity varies with d/l as shown in Fig. 7.

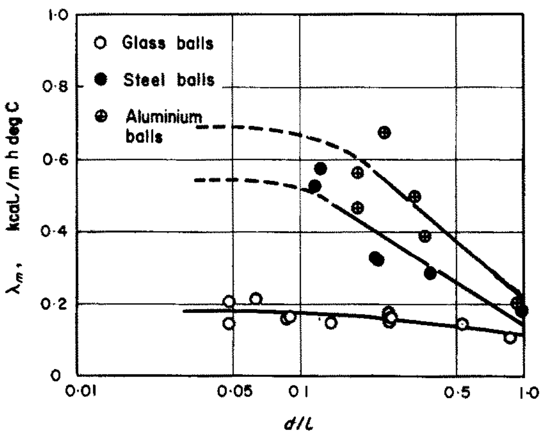

FIG. 9. Variation of thermal conductivity of porous media λ_m with d/l.

4.4 *Remarks*

Some complicated characters of the porous media have been described so far. For the present study, however, it is enough if the properties such as the permeability, porosity, and thermal diffusivity, are known respectively for every porous medium subjected to the test, and for every condition of the test. It is analogous to that, if the properties such as the density and viscosity of the fluid are known, there is no necessity to discuss the movement or structure of molecules in the ordinary fluid dynamics.

When d/l approaches unity, various factors such as the heterogeneity, anisotropy and others, inevitably arise complicating the situation from the theoretical point of view. It will be shown later, however, that the experimental results themselves do not exhibit so complex appearances even under such a special condition.

5. EXPERIMENTAL RESULTS

In the experiments for the onset of convection, nitrogen was initially stored in the pressure vessel with the maximum pressure of the gas bomb for the sake of convenience. Then, reducing the gas pressure at adequate intervals, and confirming the establishment of the steady state at each pressure, the apparent thermal conductivity of the porous layer λ was measured. The measurements were repeated a few times for every porous medium, the repetition of the same results being assured.

Typical examples of the experimental results are presented in Fig. 10 for a layer filled with glass balls, and in Fig. 11 for that of steel balls, showing the variation of λ with the gas pressure inside the layer. The convection occurs at the points where λ commences a rapid rise with the increase of the gas pressure. The reason for the gradual reduction of λ with the increase of the gas pressure in the range of the pure conduction, particularly noticeable in Fig. 11 for a packing of steel balls, has already been described in Section 4.3. This phenomenon does not give any obstruction to the present work, because the thermal conductivity of the porous medium just before the onset of convection is only necessary for the analysis, and it can be determined with ease from such experimental data as shown in Figs. 10 and 11.

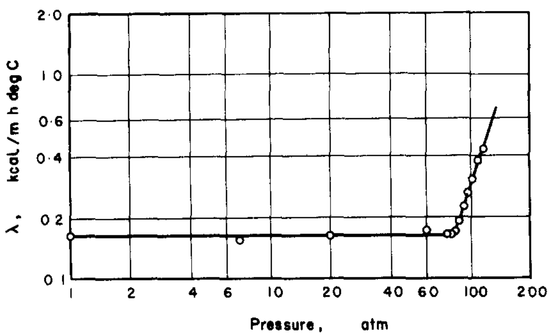

FIG. 10. Variation of apparent thermal conductivity λ with gas pressure (packing of glass balls; $d = 0{\cdot}779$ mm, $1 = 8{\cdot}9$ mm, $\Delta T = 81$ degC).

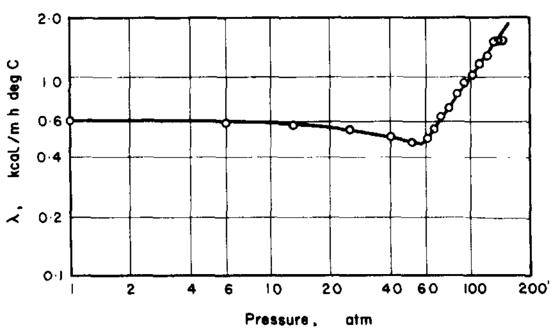

FIG. 11. Variation of apparent thermal conductivity λ with gas pressure (packing of steel balls; $d = 2{\cdot}00$ mm, $l = 17{\cdot}0$ mm, $\Delta T = 48$ degC).

5.1 *Comparisons with theoretical predictions*

In computing the critical Rayleigh number Ra from the experimental data aforementioned, all the physical properties of nitrogen are evaluated at the pressure and temperature (arithmetic mean between the two boundaries of the porous layer) of the starting point of convection. For the evaluations of the thermal expansion coefficient and density of gas, the deviations from the ideal gas law can be neglected within the present experimental range of pressure. As to the viscosity and specific heat, however, the pressure dependence is considerable so that the values corresponding to the pressure should be used.

The thermal diffusivity $\varkappa$ included in Ra is given by either (7) or (8), where the thermal conductivity of the porous medium λ_m is determined experimentally as described before. The denominator of (8): $(c_p\rho)_m$ is evaluated by

$$(c_p\rho)_m = \varepsilon(c_p\rho)_f + (1 - \varepsilon)(c_p\rho)_s$$

where ε is the porosity (measured), and the suffixes f and s represent the fluid and the material of solid fillings respectively.

In order to compare the experimental critical Rayleigh number thus obtained with the theoretical prediction given by (5), it is necessary to evaluate the permeability of the porous media k, and the following semi-empirical Blake–Kozeny equation [11] is utilized:

$$k = \frac{\varepsilon^3}{150(1 - \varepsilon)^2} d^2$$

ε denoting the porosity and d denoting the diameter of the filling particles. As is well known, this equation is valid for the laminar flow in the porous media composed with the spherical particles. In this case, (5) can be written as follows:

$$Ra \cdot \frac{\varepsilon^3}{150(1-\varepsilon)^2}\left(\frac{d}{l}\right)^2 = 4\pi^2. \tag{14}$$

Now that all is ready, let us compare the experimental results with (14). First, Fig. 12 shows the case where (7) is applied as to the thermal diffusivity $\varkappa$, and it is noticed that the agreement between the experimental results and theoretical prediction is fairly good. On the contrary, the case applying (8) as to $\varkappa$ gives Fig. 13 (N.B. the ordinates have values tenfold greater than those of Fig. 12), where not only do the experimental results heavily deviate from the theoretical prediction, but also the dispersion is slightly greater than in Fig. 12.

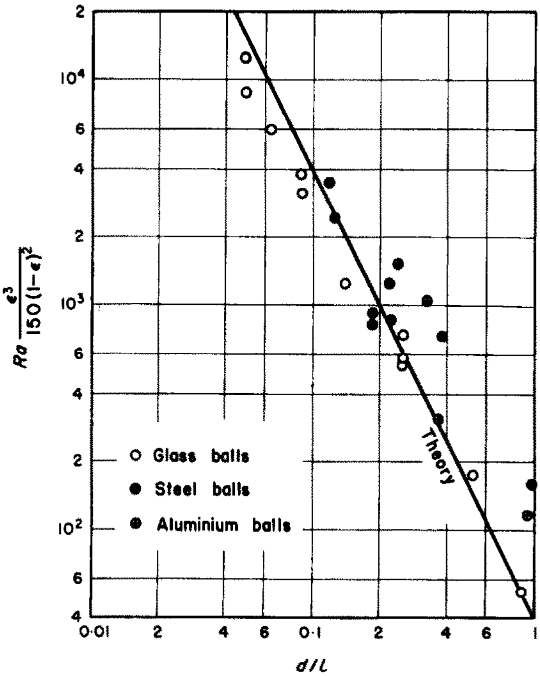

FIG. 12. Comparison of experimental critical Rayleigh number with theoretical prediction in case of $\varkappa = \lambda_m/(c_p\rho)_f$.

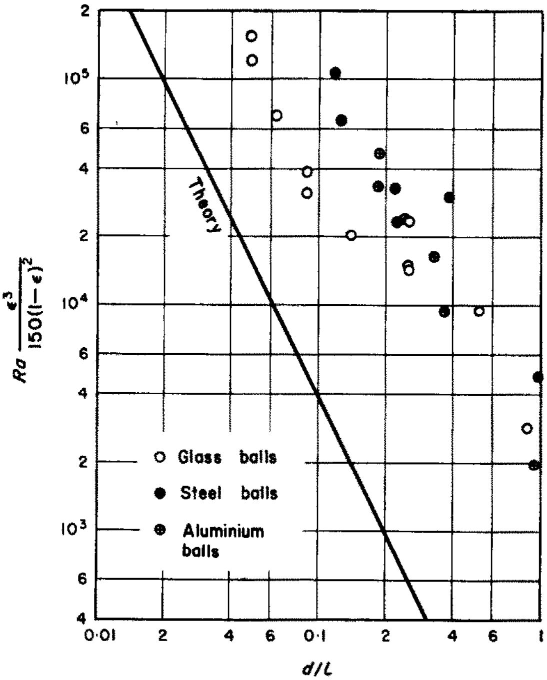

FIG. 13. Comparison of experimental critical Rayleigh number with theoretical prediction in case of $\varkappa = \lambda_m/(c_p\rho)_m$.

5.2 *Discussion*

With the result of Fig. 12, it may be concluded that the problem of the onset of convection in a porous medium has been given a conclusive answer of the same standard as in the ordinary layer of fluid, considering the inevitable unevenness of the porous media. Since the phenomenon is not essentially so complicated, it may be presumed that no peculiarity will take place even in the range of d/l less than the minimum value of d/l shown in Fig. 12 which has already reached near 0·04.

At this stage, it may be of use to point out the following. Taking the fact into account that the porosity ε usually takes values near 0·38 for small d/l, the maximum value of the ordinate of Fig. 12, that is 2×10^4, corresponds to $Ra \doteqdot 2 \times 10^7$. Approximately speaking, this is the upper limit of Rayleigh number which have been realized in the present work. Now, let us assume the case that experimental range of Ra is extended up to $6{\cdot}8 \times 10^8$, which is the maximum Ra experienced so far in the precise experiments of convection heat transfer in the ordinary horizontal layer of fluid. According to (5), however, d/l permitting the onset of convection in the porous media can be reduced only up to 0·01 or so. Of course, the convection can be generated with ease for extremely small values of d/l, provided that the thickness of the porous layer l is increased very much, but the precise experiments will become difficult as described in Section 3.1.

Finally, it should be noticed that Fig. 12 includes the data extending up to $d/l = 1$. In the range of $d/l = 0{\cdot}2$–$1{\cdot}0$ at least, the factors such as the heterogeneity, anisotropy and others should never be neglected. However, so far as the experimental data are concerned, we are given a rather simple conclusion that the criterion for the onset of convection does not heavily deviate from (5) even though d/l approaches unity.

The "range of experiment" entered in Fig. 3 corresponds to the experimental range of d/l in Fig. 12. Although there are various ambiguous problems when d/l approaches unity, it may be of interest to notice that the tendency of departure from (5) shown by the broken curve in Fig. 3 appears in the experimental data in Fig. 12 also.

6. CONCLUSIONS

(1) The criterion for the onset of convective flow in a fluid in a horizontal porous layer composed of the spherical fillings is given by

$$Ra \cdot \frac{\varepsilon^3}{150(1-\varepsilon)^2}\left(\frac{d}{l}\right)^2 = 4\pi^2,$$

where

$$Ra \equiv \frac{g\beta l^3 \Delta T}{\nu\varkappa}$$

provided that $\varkappa$ is defined as $\varkappa \equiv \lambda_m/(c_p\rho)_f$. This criterion is applicable in the range of d/l less than, say, $0{\cdot}1 \sim 0{\cdot}2$.

(2) Under special conditions near $d/l = 1$, the critical Rayleigh number shows a tendency to become higher than that given by the criterion aforementioned. However, the deviation is not so great.

(3) Judging from the characteristics of the phenomenon, it may be concluded that the general criterion for various porous layers besides the layers filled with the spherical particles is given by

$$Ra \cdot \frac{k}{l^2} = 4\pi^2$$

provided that Darcy's law can be applied for the flow in the porous media, and $\varkappa$ is defined as $\varkappa \equiv \lambda_m/(c_p\rho)_f$.

(4) After all, not only is the theoretical result of Horton and Rogers [2], and of Lapwood [3] certainly useful but also it can cover a notably wide scope, provided that the definition of the thermal diffusivity is modified.

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REFERENCES

1. A. Pellew and R. V. Southwell, On maintained convective motion in a fluid heated from below, *Proc. R. Soc.* **A176**, 312–343 (1940).
2. C. W. Horton and F. T. Rogers Jr., Convective currents in a porous medium, *J. Appl. Phys.* **16**, 367–370 (1945).
3. E. R. Lapwood, Convection of a fluid in a porous medium, *Proc. Camb. Phil. Soc.* **44**, 508–521 (1948).
4. H. L. Morrison, F. T. Rogers Jr. and C. W. Horton, Convective currents in porous media, II. Observation of conditions at onset of convection, *J. Appl. Phys.* **20**, 1027–1029 (1949).
5. F. T. Rogers Jr. and H. L. Morrison, Convection currents in porous media, III. Extended theory of the critical gradient, *J. Appl. Phys.* **21**, 1177–1180 (1950).
6. F. T. Rogers Jr., L. E. Schilberg and H. L. Morrison, Convection currents in porous media, IV. Remarks on the theory, *J. Appl. Phys.* **22**, 1476–1479 (1951).
7. M. Muskat, *Flow of Homogeneous Fluids through Porous Media*, p. 121. McGraw-Hill (1937).
8. H. Jeffreys, The stability of a layer of fluid heated below, *Phil. Mag.* **2**, 833–844 (1926).
9. P. L. Silveston, Wärmedurchgang in waagerechten

Flüssigkeitsschichten, *Forsch. Geb. IngWes.* **24**, 26–69 (1958).
10. S. Yagi, D. Kunii and N. Wakao, Radially effective thermal conductivities in packed beds, *International Developments in Heat Transfer*, pp. 742–749. ASME (1961).
11. R. B. Bird, W. E. Stewart and E. N. Lightfoot, *Transport Phenemena*, p. 196. Wiley, New York (1960).

Résumé—Le but de cet article est d'étudier soigneusement la perte de la stabilité d'une couche de fluide soumise à un gradient vertical de température dans un milieu poreux et de donner un critère décisif pour le démarrage des courants de convection.

On a mis en évidence théoriquement que le phénomène dépend d'une diffusivité thermique définie d'une façon spéciale et qu'il est possible que la théorie ordinaire basée sur la loi de Darcy soit applicable même lorsque la permeabilité du milieu poreux devient considérablement élevée. Des expériences ont également été conduites avec soin, dans lesquelles les difficultés pour engendrer l'écoulement de convection dans le milieu poreux avec un gradient de température raisonnable sont surmontées en employant un milieu poreux de perméabilité relativement élevée ainsi qu'un gaz comme fluide.

On a obtenu un accord satisfaisant entre les résultats expérimentaux et la théorie, ce qui amène à conclure que le critère pour le début de l'écoulement de convection est certainement donné par l'équation $Ra. k/l^2 = 4\pi^2$, où Ra est le nombre de Rayleigh, k la perméabilité et l l'épaisseur verticale du milieu poreux, mais que la diffusivité thermique entrant dans Ra doit être définie comme le rapport de la conductivité thermique du milieu poreux et de la chaleur spécifique du fluide.

Zusammenfassung—In der vorliegenden Arbeit wird eine sorgfältige Untersuchung durchgeführt über den Zusammenbruch der Stabilität einer Flüssigkeitsschicht in einem porösen Medium unter dem Einfluss eines vertikalen Temperaturgradienten und es wird ein abschliessendes Kriterium für das Einsetzen der Konvektionsströmung angegeben.

Auf Grund theoretischer Überlegungen wird gezeigt, dass das Phänomen von einer speziell definierten Temperaturleitfähigkeit beeinflusst wird und dass die Anwendung einer einfachen Theorie, die auf Darcy's Gesetz beruht möglich erscheint, selbst wenn die Durchlässigkeit des porösen Mediums verhälgnismässig wird. Sorgfältige Versuche wurden ebenfalls durchgeführt; darin sind die Schwierigkeiten, konvektive Ströme im porösen Medium bei tragbaren Temperaturgradienten zu erhalten dadurch umgangen, dass poröse Medien verhältnismässig grosser Durchlässigkeit und kompressible Gase als fluides Medium verwendet wurden.

Zufriedenstellende Übereinstimmung der Versuchsergebnisse mit der Theorie wird erhalten. Daraus ist zu schliessen, dass ein Kriterium für das Einsetzen der Konvektionsströmung durch die Gleichung $Ra\ k/l^2 = 4\pi^2$ gegeben wird. Dabei ist Ra die Rayleigh-zahl, k die Durchlässigkeit und l die vertikale Dicke des porösen Mediums. Die Temperaturleitfähigkeit in Ra ist definiert als die Wärmeleitfähigkeit des porösen Mediums geteilt durch die spezifische Wärme und Dichte des fluiden Stoffes.

Аннотация—Цель настоящей статьи-тщательное исследование нарушения устойчивости жидкого слоя под воздействием поперечного градиента температур в пористой среде, а также получение критерия возникновения конвективных потоков.

На основании теоретического анализа отмечается, что это явление связано с особым образом определенной температуропроводностью и что имеется возможность применения известной теории, построенной на законе Дарси, даже тогда когда проницаемость пористой системы довольно высока. В тщательных экспериментах трудность создания конвективного потока в пористой среде под действием относительно невысокого градиента температуры преодолена благодаря использованию как системы со сравнительно высокой проницаемостью, так и сжимаемого газа в качестве рабочего тела.

Удовлетворительное соответствие между теоретическими и полученными экспериментальными данными позволило сделать вывод, что критерий возникновения конвективного потока может быть представлен в виде $Ra\ .\ k/l^2 = 4\pi^2$, где Ra число Релея, k-проницаемость, l-толщина пористой системы, а температуропроводность $\varkappa$ определяется отношением теплопроводности пористой системы к удельной теплоемкости рабочего тела.